A Statistical Model for Land Mobile Satellite Channels and Its Application to Nongeostationary Orbit Systems

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***Abstract*—The paper introduces a statistical channel model which is a combination of Rice and lognormal statistics, and is suitable in principle to all types of environment (rural, suburban, urban) simply by tuning the model parameters. The model validity is confirmed by comparisons with measurement data collected in the literature. Empirical formulas are derived for the model parameters to fit measured data over a wide range of elevation angles. In particular, the model is applied to nongeostationary satellite channels, such as low-Earth orbit and medium-Earth orbit channels, in which for a given user located in a generic site the elevation angle changes continuously. Finally, examples of average bit error probability evaluations in the channel are provided.**

I. Introduction

As a consequence of the growing interest in land mobile satellite (LMS) systems, much effort is being devoted to the problem of modeling nonselective multipath fading and shadowing in the LMS communication channel. Among the different viable approaches [1], a probability distribution model, which is less complicated than a geometric analytic model and is more phenomenological than an empirical regression model, has the advantage to more easily allow performance predictions and system comparisons under different conditions of modulation, coding, and multiple access. Loo [2] proposed a model, suitable for rural environments, which assumes that the received signal is affected by nonselective Rice fading with lognormal shadowing on the direct component only, while the diffuse scattered component has constant average power level. Lutz *et al.* [3] introduced a two-state model which is Rice under good propagation conditions and Rayleigh-lognormal otherwise.

In this paper we propose a probability distribution model which is a combination of Rice and lognormal statistics, with shadowing affecting both direct and diffuse components, and not only the former as in [2]. In principle, our model is suitable to all types of environment (rural, suburban, urban) simply by tuning the model parameters. In particular, in built-up areas it reduces to the well-known and well-tested Suzuki model (i.e., Rayleigh-lognormal) [4], that is widely accepted for terrestrial land mobile macrocellular channels [5]. In this paper the model is applied to nongeostationary satellite channels, such as low-Earth orbit (LEO) and medium-Earth orbit (MEO) channels.

LEO and MEO constellations are being considered in many proposals for future LMS systems [6], with the objective to achieve a global coverage with acceptable elevation angle, α. Being such orbits nongeostationary, the elevation angle at any site changes continuously over time; as a consequence, the propagation conditions are also time varying, even if the mobile terminal is static as it is often the case in personal communications.

In order to be able to predict the performance of a system adopting such constellations, it is therefore necessary to possess a statistical model which fits measured data over a wide range of elevation angles. Here we adopt a novel approach, in which the parameters of the probability distribution model are described by empirical formulas to fit measured data. The resulting hybrid empirical-probability distribution model fits measured data over a wide range of α, and it allows the evaluation of performance characteristics such as the average bit error probability.

The paper is organized as follows. In Section II we present the statistical channel model, its validation against measured data and the empirical regression formulas for a rural tree-shadowed environment. In Section III we describe a three step procedure to evaluate the probability of error for nongeostationary systems. In Section IV we discuss some important features of the nongeostationary (in particular LEO and MEO) satellite channels and then provide examples of bit error probability evaluations. In Section V we draw paper conclusions.

II. Statistical Channel Modeling and Validation

A. Probability Distribution Model

The probability density function (p.d.f.) of received signal envelope, r, is given by

$$p_r(r) = \int_0^\infty p(r|S)p_S(S)\,dS. \qquad (1)$$

In (1) $p(r|S)$ is a Rice p.d.f. conditioned on a certain

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shadowing, S:

$$p(r|S) = 2(K+1)\frac{r}{S^2}\exp\left[-(K+1)\frac{r^2}{S^2} - K\right] \cdot I_0\left(2\frac{r}{S}\sqrt{K(K+1)}\right) \quad (r \geq 0) \tag{2}$$

where I_0 is the zero order modified Bessel function of the first kind, and K is the so called Rice factor. The shadowing, S, is lognormal with p.d.f.:

$$p_S(S) = \frac{1}{\sqrt{2\pi}\,h\sigma S}\exp\left[-\frac{1}{2}\left(\frac{\ln S - \mu}{h\sigma}\right)^2\right] \quad (S \geq 0) \tag{3}$$

where $h = (\ln 10)/20$, μ and $(h\sigma)^2$ are the mean and the variance of the associated normal variate, respectively; in terrestrial channels σ is usually referred to as the "dB spread."

When $K = 0$, (1)-(3) provide the Suzuki p.d.f. In the limit for $\sigma \to 0$, $p_S(S)$ tends to a Dirac pulse located at the mean value of the distribution, i.e., it tends to $\delta(S - e^{\mu})$. Therefore $p_r(r) \to p(r|e^{\mu})$ and the channel is Rice.

The signal envelope which meets the channel model (1)-(3) can be interpreted as the product of two independent processes, i.e., $r = RS$, where R is a Rice process, and S is lognormal. Due to the independence between R and S we have [7]:

$$p_r(r) = \int_0^\infty \frac{1}{S}p_R\left(\frac{r}{S}\right)p_S(S)\,dS = \int_0^\infty \frac{1}{R}p_S\left(\frac{r}{R}\right)p_R(R)\,dR \tag{4}$$

and by comparing (1) and (4):

$$p(r|S) = \frac{1}{S}p_R\left(\frac{r}{S}\right) \triangleq \frac{r}{\sigma_R^2 S^2} \cdot \exp\left[-\frac{1}{2}\left(\frac{r^2}{S^2\sigma_R^2} + 2K\right)\right] \cdot I_0\left(\frac{r}{S\sigma_R}\sqrt{2K}\right) \quad (r \geq 0) \tag{5}$$

which implies $\sigma_R^2 = 1/2(K+1)$. Equation (4) allows further observations: when $K \to \infty$, $p_R(R)$ tends to a Dirac pulse located at $R = 1$ and $p_r(r)$ tends to $p_S(r)$, i.e., the channel is lognormal. When $K \to \infty$ and $\sigma \to 0$ fading is absent. Therefore, depending on the combination of K, μ, σ the proposed channel model can be reduced to any one of the usual nonselective fading models.

From the moments of the lognormal process [8] and those of the Rice process [9], the nth order moment of r can be derived as

$$E\{r^n\} = E\{R^n\}E\{S^n\} = (K+1)^{-n/2}e^{-K}\Gamma\left(1+\frac{n}{2}\right) \cdot e^{n\mu}\exp\left(\frac{1}{2}n^2h^2\sigma^2\right) \cdot {}_1F_1\left(1+\frac{n}{2}, 1; K\right) \tag{6}$$

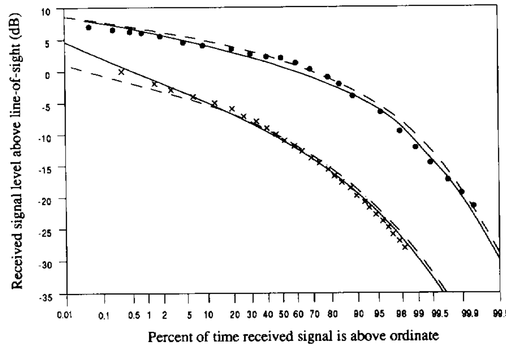

Fig. 1. Comparison between measured c.d.f. data in light shadowing (●) and heavy shadowing (×), the c.d.f. given by (7) (continuous lines), and the c.d.f. provided in [2] (dashed lines). Light shadowing: $K = 4.0$, $\mu = 0.13$, $\sigma = 1.0$. Heavy shadowing: $K = 0.6$, $\mu = -1.08$, $\sigma = 2.5$.

where Γ is the gamma function and ${}_1F_1$ is the confluent hypergeometric function [10].

Finally, the cumulative distribution function (c.d.f.) of the envelope is

$$P_r(r_0) \triangleq \mathrm{Prob}\{r < r_0\} = \int_0^{r_0}\int_0^\infty \frac{p_S(S)}{S}p_R\left(\frac{r}{S}\right)dS\,dr = 1 - E_S\left\{Q\left(\sqrt{2K}, \frac{r_0}{S}\sqrt{2(K+1)}\right)\right\} \tag{7}$$

where $E_S\{\ \}$ stands for the average with respect to S and Q is the Marcum's Q-function [9].

B. Model Validation and Empirical Formulas

The proposed channel model was validated with respect to measurement data available in the literature. Fig. 1 collects the c.d.f. measured data provided in [2] for the cases referred to as "infrequent light shadowing" and "frequent heavy shadowing." In the same Fig. 1 we provide the fitting curves obtained by means of (7) with parameters μ, σ, K optimized by trial and error, and for comparison the curves calculated with the c.d.f. in [2]. Curves calculated with our model match very well both with measurement data and with Loo's c.d.f.

As stated in the "Introduction," empirical formulas should be derived to fit measured data over a wide range of elevation angles. As an example, we used some data collected by ESA at L-band in a rural tree-shadowed environment [11]. Data fitting was conditioned on the following intuitive indications: the greater is α, the larger is K and the smaller is σ. The resulting empirical formulas allow interpolation for any α in the range $20° < \alpha < 80°$:

$$K(\alpha) = K_0 + K_1\alpha + K_2\alpha^2$$
$$\mu(\alpha) = \mu_0 + \mu_1\alpha + \mu_2\alpha^2 + \mu_3\alpha^3$$
$$\sigma(\alpha) = \sigma_0 + \sigma_1\alpha. \tag{8}$$

TABLE I
COEFFICIENTS FOR EMPIRICAL FORMULAS (RURAL TREE-SHADOWED ENVIRONMENT)

K	μ	σ
$K_0 = 2.731$ $K_1 = -1.074\ 10^{-1}$ $K_2 = 2.774\ 10^{-3}$	$\mu_0 = -2.331$ $\mu_1 = 1.142\ 10^{-1}$ $\mu_2 = -1.939\ 10^{-3}$ $\mu_3 = 1.094\ 10^{-5}$	$\sigma_0 = 4.5$ $\sigma_1 = -0.05$

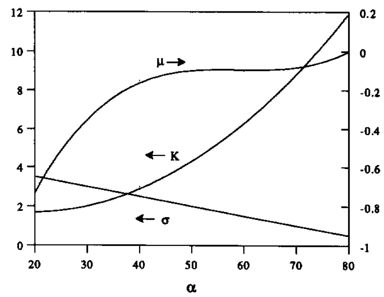

Fig. 2. Model parameters, μ, σ, and K as a function of the elevation angle, α, in a rual tree-shadowed environment.

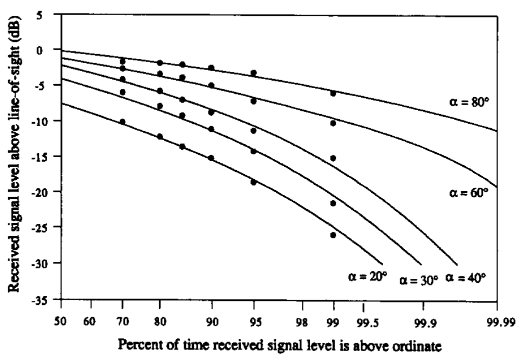

Fig. 3. Comparison between measured c.d.f. data in a rural tree-shadowed environment (●) as a function of the elevation angle, α, and the proposed c.d.f. with parameters given by (8) (continuous lines).

The coefficients for the specific example are reported in Table I and the resulting curves for K, μ and σ are shown in Fig. 2, while measured data fitting is reported in Fig. 3.

III. Probability of Error in the Nongeostationary LMS Channel

The symbol error probability for transmission in channels affected by time and frequency nonselective fading can be written as

$$P_e = \int_0^\infty P(e|r)p_r(r)\,dr \tag{9}$$

where $P(e|r)$ is the symbol error probability conditioned on a certain value of r and $p_r(r)$ is given by (4). By substituting (4) into (9) and interchanging the order of integration we have

$$P_e = \int_0^\infty p_S(S)\left[\int_0^\infty P(e|RS)p_R(R)\,dR\right]dS \tag{10}$$

where $R = r/S$. The inner integral represents the average error probability in the presence of Rice fading only (i.e., for a given value of S):

$$E_R\{P(e|RS)\} = \int_0^\infty P(e|RS)p_R(R)\,dR \triangleq f(S) \tag{11}$$

while the outer integral in (10) is the average of $f(S)$ in the presence of lognormal shadowing. In conclusion, we have:

$$P_e = E_S\{f(S)\} = E_S\{E_R[P(e|r)]\}. \tag{12}$$

The error probability provided by (12) depends on the model parameters μ, σ and K, which for a given site are functions of the elevation angle, α.

While for geostationary systems (12) itself provides the local average probability of error, in the case of nongeostationary orbit systems P_e must be additionally averaged with respect to the angle p.d.f.:

$$\overline{P}_e = E_\alpha\{P_e\}. \tag{13}$$

The three step procedure given by (11)-(13) for evaluating the error probability is applied in the following section to some specific examples.

IV. Numerical Results for Some Nongeostationary Orbit Systems

With the aim of characterizing nongeostationary orbit systems, a software package was developed. The package simulates the satellite constellation motion around the Earth, and evaluates system parameters and functions such as instantaneous coverage, satellite handovers, link unavailabilty, etc. [12]. In particular, the c.d.f. of the elevation angle can be achieved under minimum, average, and maximum global coverage conditions. As an example, Fig. 4 shows the minimum worldwide coverage, C_m (%), as a function of α for some LEO and MEO constellations [6]. We note that the fractional area covered with high-elevation angles, e.g., $\alpha \geq 60°$, is generally small. This means that to achieve adequate protection from shadowing nongeostationary communication systems must be provided with large power margins. From a channel modeling point of view, we must resort to local statistics for the elevation angle. For instance, setting Roma as the location in which the performance needs to be evaluated, the corresponding discretized p.d.f. for α is reported in Table II for the different constellations considered.

Making use of the proposed model, the bit error probability for binary DPSK modulation has been evaluated at different elevation angles (see Fig. 5). For high elevation angles (greater than 60°), when shadowing is negligible, it is evident an error floor in the bit error probability curves

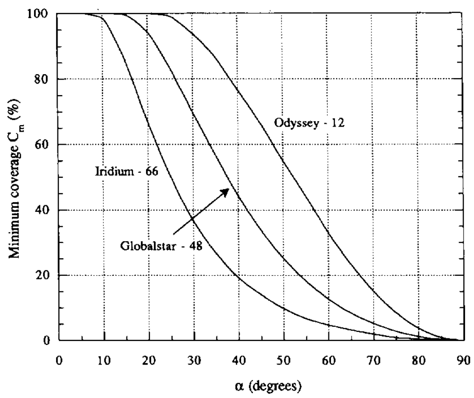

Fig. 4. Minimum global coverage (polar regions excluded) of some LEO and MEO systems as a function of the elevation angle, α (the number following the system name provides the constellation size).

TABLE II
DISCRETIZED PROBABILITY DENSITY FUNCTION FOR THE ELEVATION ANGLE, % (SITE: ROMA; 41.9 N LAT., 12.5 E LONG)

α (°)	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90
A: Globalstar-48	0.0	0.0	0.0	21.9	44.0	22.7	8.4	2.6	0.4
B: Iridium-66	0.0	16.5	32.1	26.3	11.6	6.9	4.2	1.6	0.8
C: Odyssey-12	0.0	0.0	5.2	15.1	19.6	19.0	19.4	16.9	4.8

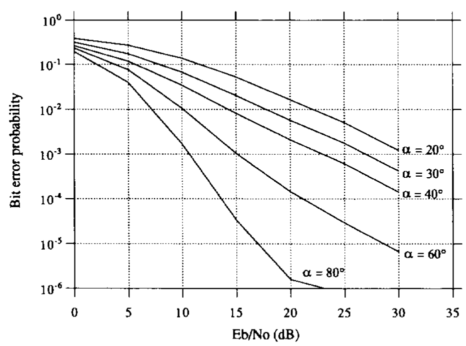

Fig. 5. Bit error probability, P_e, for DPSK modulation as a function of the bit energy/noise power spectral density, E_b/N_0, for several values of the elevation angle, α.

due to the presence of the diffuse component. Averaging the bit error probability over the discretized range of elevation angles produces the average bit error probability for the different constellations as reported in Fig. 6. Obviously, these results cannot be used to compare the actual systems for several reasons, among which: DPSK is not the adopted modulation scheme; the systems have different orbital heights, so that free-space losses are much different; shadowing and multipath fading are not the only impairments in the channel (in particular, co-channel interference has to be considered). Furthermore, the results strongly depend on latitude.

The presented approach to the estimation of the average bit error probability provides a tool that enhances the link

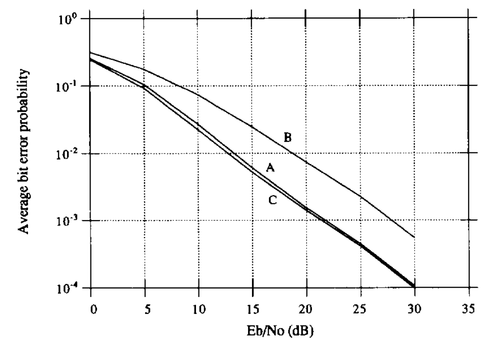

Fig. 6. Average bit error probability, $\overline{P}_e$, for DPSK modulation as a function of the bit energy/noise power spectral density, E_b/N_0, associated to the probability density functions for the elevation angle given in Table II (site: Roma).

budget accuracy of any system adopting nongeostationary orbits.

V. CONCLUSIONS

The paper described a statistical model for land mobile satellite communications that is suitable to different propagation environments and a wide range of elevation angles. Therefore, the model is proposed for the statistical characterization of LEO and MEO satellite communications. The model allows to predict communication performance under different conditions of modulation, coding, and multiple access. The paper showed this by evaluating the average probability of error in few simple cases.

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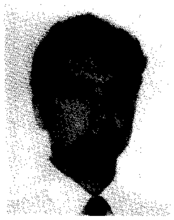

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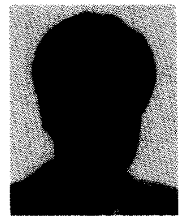

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